

TABLE I

Type	Material	Specific Resistance (ohm-cm)	Holder	Commercial or Experimental
GSB1	Ge	0.05	1N263 Type	Commercial
GSB2	Ge	0.1	1N263 Type	Commercial
SISBR	Si	0.03	1N263 Type	Experimental
SISBV	Si	0.1-0.08	1N263 Type	Experimental

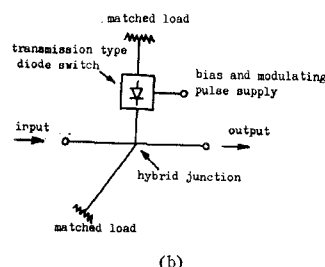
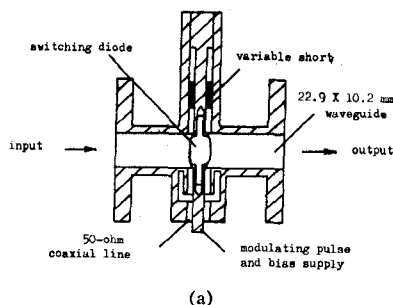


Fig. 1—Construction of diode switches.

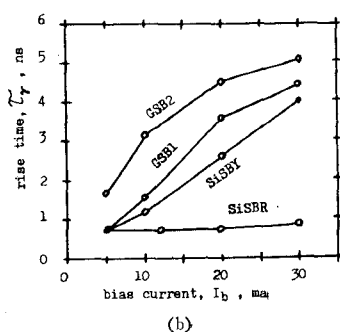
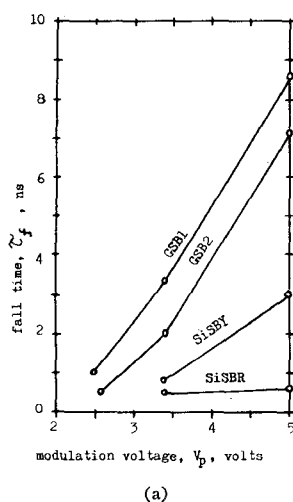
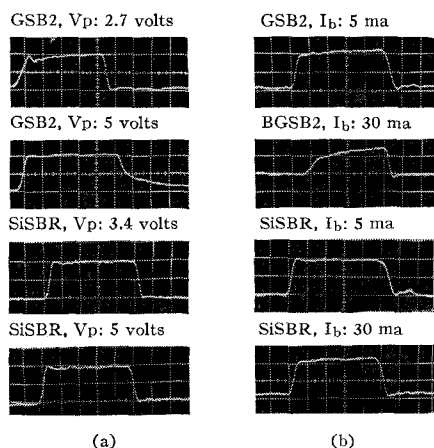


Fig. 2—Rise and fall times of diode switches in the conducting case. (a) Transmission type (microwave input power = 5 mw, static bias voltage = -5 v). (b) Reflection type (microwave input power = 5 mw, applied pulse voltage = 2 v).

Fig. 3—Microwave pulse waveforms (in-phase components) of diode switch output. V_p = applied pulse voltage, I_b = static bias current, horizontal axis = 2 nsec/div. (a) Transmission type (microwave input power = 5 mw, static bias voltage = -5 v). (b) Reflection type (microwave input power = 5 mw, applied pulse voltage = 2 v).

the hole storage effect. The switching fall time was almost independent of the static bias current and was about 0.5-0.7 nsec.

Fig. 3 illustrates the typical waveforms (in-phase components) of switched microwave pulses when the GSB2 and SiSB diodes are applied with a rectangular pulse of 20 nsec in the transmission and reflection-type switches.

It was concluded that the silicon silver-bonded diode SiSB has the best rise and fall times: about 0.5 nsec.

ACKNOWLEDGMENT

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Addendum to "An Exact Method for Synthesis of Microwave Band-Stop Filters"

SUMMARY

An algorithm is presented that enables a designer to compute efficiently and quickly the design parameters of microwave band-stop filters that are based on the exact synthesis technique previously discussed by Schiffman and Matthaei. Tables of band-stop filter designs that give maximally flat and Chebyshev responses are presented for ten typical stop-band fractional bandwidths.

INTRODUCTION

By extending work of Ozaki and Ishii [1], Schiffman and Matthaei [2], [3] have presented a theory and several design equations for an exact method of synthesizing microwave band-stop filters. Using their synthesis procedure, a band-stop filter is developed from a low-pass, lumped-element prototype filter. The band-stop filter is composed of either quarter-wavelength short-circuited stubs connected in series with the main transmission line, or quarter-wavelength open-circuited stubs connected in shunt with the main transmission line. The stubs are spaced at either quarter- or three-quarter-wavelengths. The characteristic immittances of the stubs and connecting lines depend on the low-pass filter that is used as a prototype. If the low-pass prototype filter has an attenuation function $L_A(\omega'/\omega_1')$, the band-stop filter will have a response $L_A\{\Delta \tan[(\pi/2)(\omega/\omega_0)]\}$ where L_A is the transducer attenuation in decibels. The arguments of the attenuation function L_A are defined as follows:

- ω' is the frequency variable of the low-pass prototype filter,
- ω_1' is the cutoff frequency of the low-pass prototype filter,
- ω is the frequency variable of the band-stop filter,
- ω_0 is the center frequency of the band-stop filter,
- Δ is a scaling parameter that is defined

$$\Delta = \omega_1' \tan[(\pi/4)w]. \quad (1)$$

The symbol w in (1) is the fractional stop bandwidth of the band-stop filter. It is defined by

$$w = \frac{\omega_2 - \omega_1}{\omega_0}, \quad (2)$$

where ω_2 and ω_1 are the cutoff frequencies of the band-stop filter corresponding to ω_1' of the low-pass prototype filter.

Figs. 1(a) and 1(b) show configurations of the low-pass prototype filter used in the synthesis procedure. Note that the parameters g_i of the prototype filter are also defined in Figs. 1(a) and 1(b). Figs. 1(c) and 1(d) show typical maximally flat and Chebyshev responses of the filters of Figs. 1(a) and 1(b), respectively. Tables of g_i

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that give maximally flat or Chebyshev responses are presented in the literature [4], [5]. Figs. 2(a) and 2(b) show the derived band-stop filter using short-circuited and open-circuited stubs, respectively. The filters of Figs. 2(a) and 2(b) use quarter-wavelength connecting lines, while in Figs. 3(a) and 3(b), page 372, the filters are shown using three quarter-wavelength connecting lines. Note also that in Figs. 2(a), 2(b), 3(a) and 3(b), the parameters h_i , h_{i+1} and $(h_{i+1})_j$ are defined. These parameters are characteristic impedances or admittances of the transmission lines. They correspond to the g_i of the lumped-prototype filter. Fig. 4 shows an idealized response of the bandstop filter when designed from a Chebyshev prototype. The steady-state response would be the same for all the filters shown in Figs. 2 and 3.

In [2] and [3], equations are given for the cases of $n=1$ through 5 resonators, that relate the g_i of the low-pass prototype filter to the h_i and h_{i+1} of the band-stop filter with quarter-wavelength spacing between stubs. Also, equations are presented for $n=1, 2$ and 3 , relating the g_i of the prototype filter to the h_i and $(h_{i+1})_j$ of the band-stop filter with three-quarter wavelength spacing. Although it is desirable to have expressions for larger values of n , they have not been developed because the algebra is unwieldy. Also, (particularly for filters with three-quarter wavelength spacing between stubs), the formulas for large n would probably require longer computational time than the method that will be presented in this communication. For these reasons, an algorithm that systematically calculates the h parameters of the band-stop filter has been developed. The algorithm is a twofold iterative procedure that can be applied to a low-pass prototype filter of the type shown in Fig. 1 having any number of g 's. When the algorithm is used in conjunction with a desk calculator, most filter designs can be computed in 30 minutes or less, depending on the magnitude of n . (The first design example presented in this communication has $n=8$, and was computed in less than 25 minutes on a Friden desk calculator.)

AN ALGORITHM FOR THE h PARAMETERS OF EXACT-DESIGNED BAND-STOP FILTERS

The algorithm is best explained by presenting two examples. The first example presents the computation of the h parameters of a band-stop filter that has eight stubs separated by quarter-wavelength lines. The filter has a 50 per cent stop-band bandwidth. It is based on an $n=8$, 0.5-db ripple, Chebyshev prototype filter. The g_i of the prototype filter are given in Table I [5].

To compute the h 's, the g_i are first modified by multiplying g_i through g_n (but not g_0 or g_{n+1}) by Λ , where recall that $\Lambda = \omega_1' \tan[(\pi/4)w]$. The modified g_i will be denoted as g_i' . They are given in Table II, page 373. Next, a computation matrix is begun in the manner shown in Fig. 5(a). Referring to Fig. 5(a), note that the number of (g_i') 's to be placed in the zero column (the first column on the left) is given by the

value of the parameter P . The following rules are used to determine the value of P : if n is odd,

$$P = \frac{n}{2} - \frac{1}{2}; \quad (3)$$

if n is even,

$$P = \begin{cases} \frac{n}{2} - 1, & \text{in the development of the} \\ & \text{first half of the filter} \\ \frac{n}{2}, & \text{in the development of the} \\ & \text{second half of the filter.} \end{cases} \quad (4)$$

In the example being worked, n is 8. Therefore, $P=(8/2)-1=3$ for the development of the first half of the filter. Next, we proceed to compute the entries in the matrix. To compute an entry in an odd-numbered column, use the rule shown in Fig. 5(b). For example, the entry in the square obtained from the intersection of the row and column marked 1 is given by $[1/(0+1)]=1$. The remaining entries in column 1 are computed in the same way and appear in Fig. 5(c).

To compute an entry in an even-numbered column, use the rule shown in Fig. 5(d). For example, the entry in the square obtained from the intersection of the row marked 1 and the column marked 2 is given by $[(1 \times 0)/1]=0$. The remaining entries in column 2 are computed in the same way and appear in Fig. 5(e). Next, using the appropriate rule for each column, the remaining entries of the matrix are computed. The result is given in Fig. 5(f).

The h parameters appear in the last row of the matrix as shown in Fig. 5(f). The terminating parameter corresponding to the source immittance is denoted by h_0 . Its value appears in the last row and column. Adjacent to it is the first stub parameter, h_1 . Adjacent to that is the connecting line parameter, h_{12} . Adjacent to that is a stub, etc.

If the band-stop filter is to be composed of short-circuited series stubs, the h parameters are in ohms. If the band-stop filter is to be composed of open-circuited shunt stubs, the h parameters are in mhos.

To calculate the h parameters of the second half of the band-stop filter, we use the preceding computational method but with g' values from the second half of the prototype filter. The computation matrix is begun as shown in Fig. 6(a), page 374. In this case, $P=(8/2)=4$, as given by (4). The entries in the matrix are computed using the same rules as before. The completed matrix is given in Fig. 6(b).

The reader may notice that the value of the parameter h_4 has not been computed. It is given by the value of g_4' . In general, the value of the immittance of the "middle" stub is given by its g' counterpart. The following rules are a useful reminder.

$$\text{For } n \text{ even, } h_{n/2} = g'_{n/2}. \quad (5)$$

$$\text{For } n \text{ odd, } h_{n/2+1/2} = g'_{n/2+1/2}. \quad (6)$$

To calculate the h parameters of a band-stop filter with three-quarter wavelength spacing between stubs, use the previously explained algorithm but note the following

amendment. The initial setup of the computation matrix is modified as follows. In applying (3) or (4) to determine the parameter P , each g_i' entry in the zero column of the computation matrix is followed by two zero entries. This procedure is demonstrated by the worked example given in Fig. 7(a) and 7(b), page 375.

The example given in Figs. 7(a) and 7(b) is for an $n=4$, 0.5-db ripple, Chebyshev band-stop filter having a 50 per cent stop-band bandwidth. The g_i and g_i' values are given in Tables III and IV, respectively [5]. Note in Figs. 7(a) and 7(b) that the effect of introducing zeros in the initial column of the matrix is to produce "dummy stubs" of zero characteristic immittance. These stubs are ignored in the synthesis of the filter. Note also that the indexing of the connecting lines is reversed in the computation matrix of the second half of the filter as compared to the indexing in the first half of the filter. This gives indexes for the final structure, which increase continuously from one end of the filter to the other.

As in the previous example, $h_{n/2}$ is given by $g'_{n/2}$ according to (5).

TABLES OF h PARAMETERS FOR MAXIMALLY FLAT AND CHEBYSHEV BAND-STOP FILTERS

The algorithm previously given was programmed on an electronic digital computer and several tables of h 's (Tables V-IX, pages 376-382) compiled for maximally flat and Chebyshev band-stop filters having various fractional bandwidths w .

For n odd, only one half of the h parameters are presented in the tables because the filters are symmetrical about the center stub. In these cases, the h parameters obey the relationship

$$h_{n+1-i} = h_i \quad (7)$$

$$h_{n-i,n-i+1} = h_{i,i+1}. \quad (8)$$

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The computer programming of the algorithm for the tables of band-stop filter designs was done by P. H. Omlor. P. Rezek calculated the examples presented in the text.

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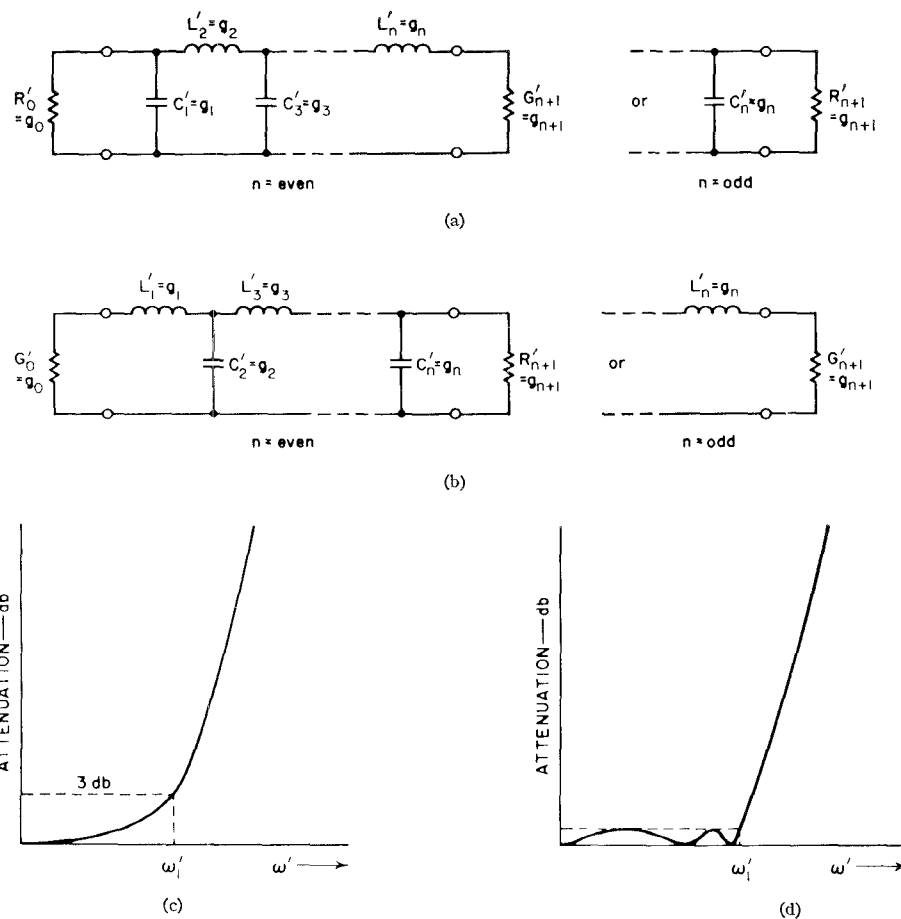


Fig. 1—Low-pass prototype filter: four basic circuit types defining the parameters g_0, g_1, \dots, g_{n+1} and maximally flat and equiripple characteristics defining the band edge ω'_i .

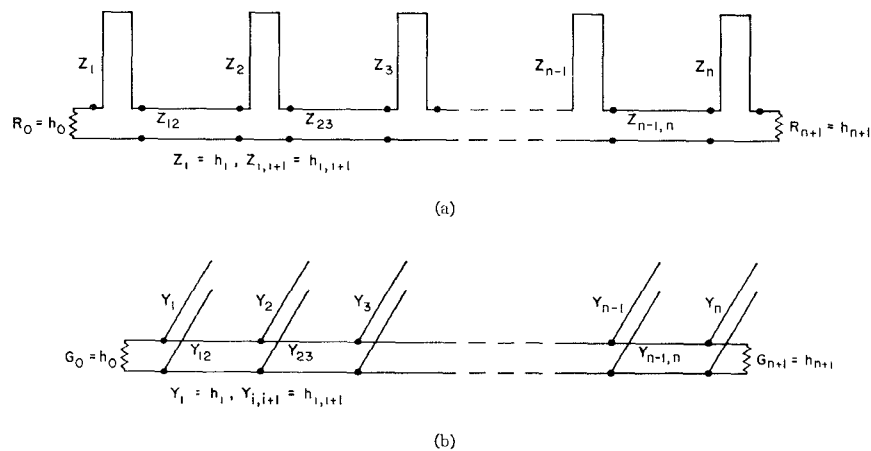


Fig. 2—Two dual band-stop filter structures using $\lambda_0/4$ stubs and $\lambda_0/4$ connecting lines, and definitions of h parameters. (a) Short-circuited stubs. (b) Open-circuited stubs.

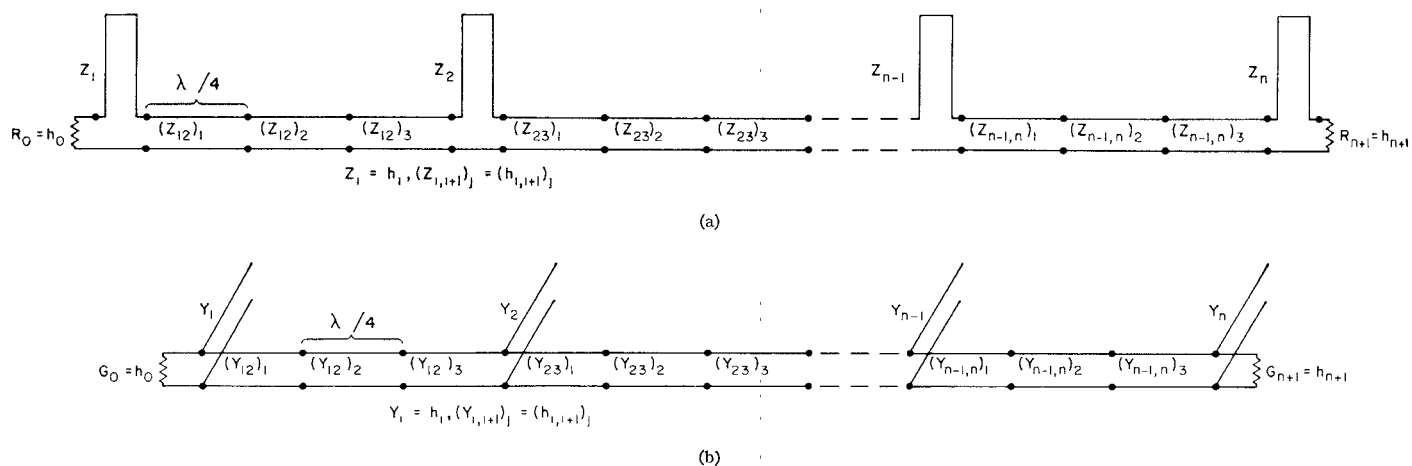


Fig. 3—Two dual band-stop filter structures using $\lambda_0/4$ stubs and $3\lambda_0/4$ connecting lines, and definitions of h parameters.
(a) Short-circuited stubs. (b) Opencircuited stubs.

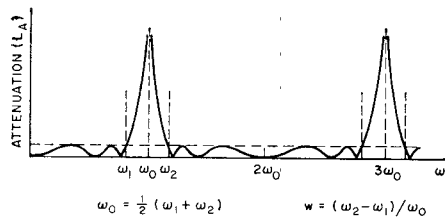


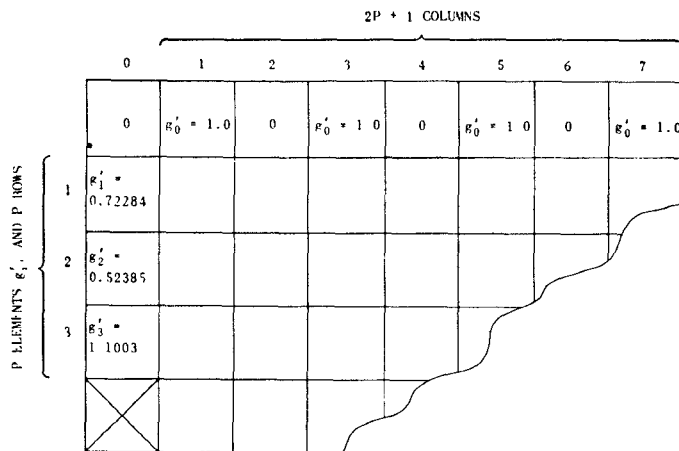
Fig. 4—Typical equiripple idealized response of band-stop filters designed on the exact basis.

TABLE I
g VALUES FOR AN $n=8$, 0.5-db CHEBYSHEV PROTOTYPE FILTER

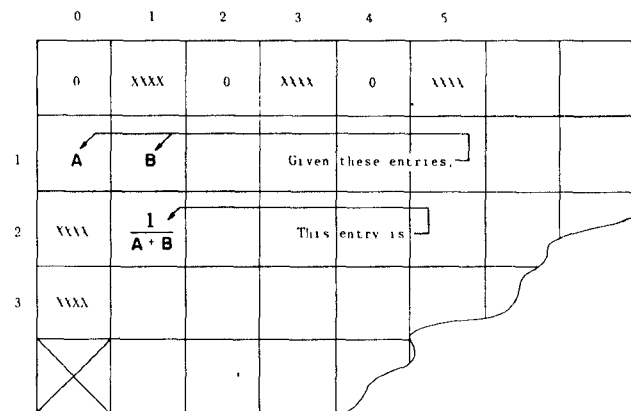
g_0	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9
1.0	1.7451	1.2647	2.6564	1.3590	2.6964	1.3389	2.5093	0.8796	1.9841

TABLE II
 g' VALUES USED IN THE FIRST EXAMPLE IN THE TEXT
 $g'_1 = \Delta g_1$
 $\omega_1' = 1.0$

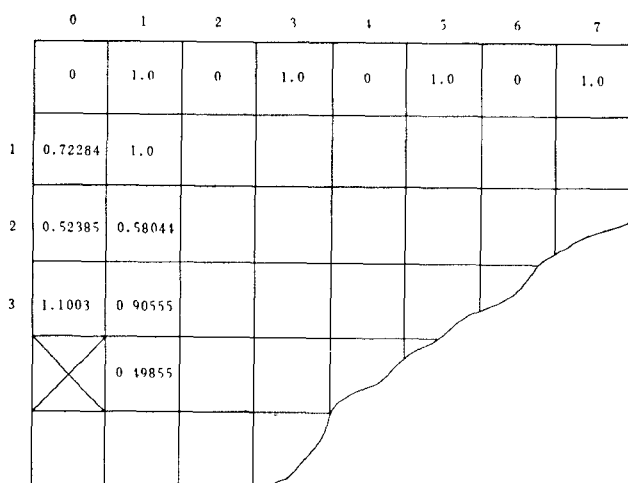
g'_0	g'_1	g'_2	g'_3	g'_4	g'_5	g'_6	g'_7	g'_8	g'_9
1.0	0.72284	0.52385	1.1003	0.56291	1.1169	0.55458	1.0394	0.36434	1.9841



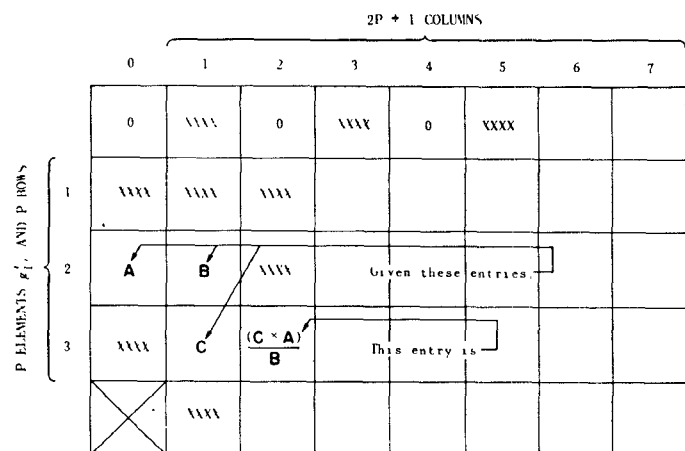
(a)



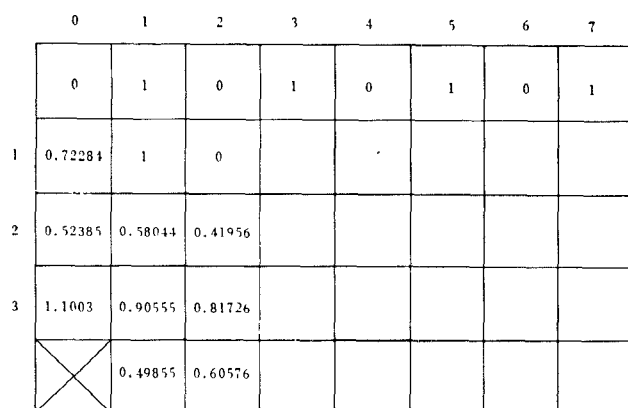
(b)



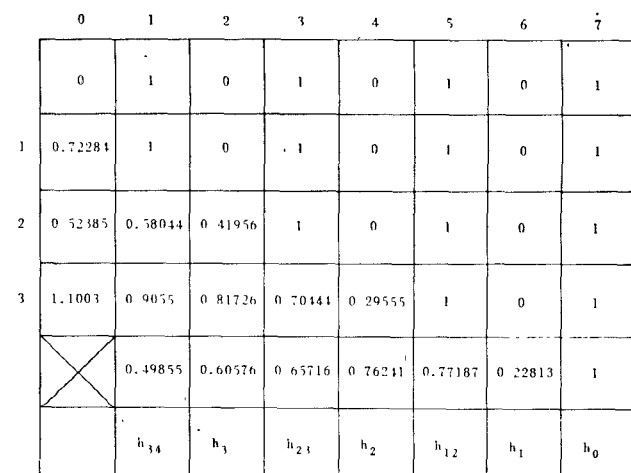
(c)



(d)




(e)



(f)


Fig. 5—Example calculation of the h parameters of a band-stop 0.5-db ripple Chebyshev filter having eight stubs with quarter-wavelength spacing between stubs (first half of filter). (a) Start of computation matrix. (b) Rule for computing entries in odd-numbered columns. (c) Example calculation of column 1. (d) Rule for computing entries in even-numbered columns. (e) Example calculation of column 2. (f) Completed computation matrix.

2P + 1 COLUMNS

	0	1	2	3	4	5	6	7	8	9
	0	$\epsilon'_{n+1} = 1.9841$	0	$\epsilon'_{n+1} = 1.9841$	0	$\epsilon'_{n+1} = 1.9841$	0	$\epsilon'_{n+1} = 1.9841$	0	$\epsilon'_{n+1} = 1.9841$
1	$\epsilon'_n = \epsilon'_8 = 0.36434$									
2	$\epsilon'_{n-1} = \epsilon'_7 = 1.0394$									
3	$\epsilon'_{n-2} = \epsilon'_6 = 0.55458$									
4	$\epsilon'_{n-3} = \epsilon'_5 = 1.1169$									
										

P ELEMENTS ϵ'_1 AND P ROWS

a)

	0	1	2	3	4	5	6	7	8	9
	0	1.9841	0	1.9841	0	1.9841	0	1.9841	0	1.9841
1	0.36434	0.50401	0	0.50401	0	0.50401	0	0.50401	0	0.50401
2	1.0394	1.1516	0.83216	1.9841	0	1.9841	0	1.9841	0	1.9841
3	0.55458	0.45641	0.41194	0.35504	0.14896	0.50401	0	0.50401	0	0.50401
4	1.1169	0.98912	1.2019	1.3038	1.5128	1.5315	0.45623	1.9841	0	1.9841
		0.47483	0.53617	0.39909	0.36790	0.32848	0.32447	0.41039	0.09362	0.50401
		h_{45}	h_5	h_{56}	h_6	h_{67}	h_7	h_{78}	h_8	h_9

(b)

Fig. 6—Example calculation of the h parameters of a band-stop 0.5-db ripple Chebyshev filter having eight stubs with quarter-wavelength spacing between stubs (second half of filter).

		2(3P) + 1 COLUMNS							
		1	2	3	4	5	6	7	
P ELEMENTS g'_1 , AND 3P ROWS		0	1	0	1	0	1	0	1
	1	0.69185	1	0	1	0	1	0	1
	2	0	0.59107	0.40893	1	0	1	0	1
	3	0	1.69185	0	0.70976	0.29024	1	0	1
		X	0.59107	0	1.40893	0	0.77505	0.22495	1
			$(h_{12})_3$	DUMMY STUB	$(h_{12})_2$	DUMMY STUB	$(h_{12})_1$	h_1	h_0

(a)

		2(3P) + 1 COLUMNS													
		1	2	3	4	5	6	7	8	9	10	11	12	13	
P ELEMENTS g'_1 , AND 3P ROWS		0	1.9841	0	1.9841	0	1.9841	0	1.9841	0	1.9841	0	1.9841	0	
	1	0.34872	0.50401	0	0.50401	0	0.50401	0	0.50401	0	0.50401	0	0.50401	0	
	2	0	1.1727	0.81138	1.9841	0	1.9841	0	1.9841	0	1.9841	0	1.9841	0	
	3	0	0.85273	0	0.35772	0.14629	0.50401	0	0.50401	0	0.50401	0	0.50401	0	
	4	0.98006	1.1727	0	2.7955	0	1.5378	0.44634	1.9841	0	1.9841	0	1.9841	0	
	5	0	0.4652	0.38821	0.35772	0	0.6503	0	0.41145	0.09256	0.50401	0	0.50401	0	
	6	0	2.1527	0	1.3406	1.45486	1.5378	0	2.4304	0	1.67626	0.30782	1.9841	0	
		X	0.46452	0	0.74593	0	0.33416	0.31614	0.41145	0	0.59660	0	0.43632	0.06769	
			$(h_{23})_1$	DUMMY STUB	$(h_{23})_2$	DUMMY STUB	$(h_{23})_3$	h_3	$(h_{31})_1$	DUMMY STUB	$(h_{34})_2$	DUMMY STUB	$(h_{34})_3$	h_4	h_5

(b)

Fig. 7—(a) Example calculation of the h parameters of a band-stop 0.5-db ripple Chebyshev filter having four stubs with three-quarter wavelength spacing between stubs (first half of filter). (b) Example calculation of the h parameters of a band-stop 0.5-db ripple Chebyshev filter having four stubs with three-quarter wavelength spacing between stubs (second half of filter).

TABLE III
 g VALUES FOR AN $n=4$, 0.5-db CHEBYSHEV
PROTOTYPE FILTER

g_0	g_1	g_2	g_3	g_4	g_5
1.0	1.6703	1.1926	2.3661	0.8419	1.9841

TABLE IV
 g' VALUES USED IN THE SECOND EXAMPLE
IN THE TEXT

g'_0	g'_1	g'_2	g'_3	g'_4	g'_5
1.0	0.69185	0.49399	0.98006	0.34872	1.9841

TABLE V
 h VALUES FOR MAXIMALLY FLAT FILTERS HAVING h_0 AND $h_{n+1} = 1.0$ AND VARIOUS STOP-BAND 3-DB FRACTIONAL BANDWIDTHS w

$n = 2$										
w	0.02	0.05	0.10	0.20	0.30	0.50	0.70	1.00	1.40	1.80
h_1	.02221	.05556	.11128	.22396	.33947	.58570	.86650	1.4140	2.7751	8.9276
h_{12}	.97827	.94737	.89986	.81702	.74656	.63064	.53576	.41425	.26489	.10073
h_2	.02173	.05263	.10014	.18298	.25344	.36936	.46424	.58575	.73511	.89927

$h_{n+1-i} = h_i$ $h_{n-i, n-i+1} = h_{i, i+1}$ $n = 3$										
w	0.02	0.05	0.10	0.20	0.30	0.50	0.70	1.00	1.40	1.80
h_1	.01547	.03780	.07296	.13673	.19360	.29289	.37996	.50000	.66246	.86327
h_{12}	.98453	.96220	.92704	.86327	.80640	.70711	.62004	.50000	.33754	.13673
h_2	.03142	.07858	.15740	.31677	.48016	.82843	1.2256	2.0000	3.9252	12.628

$n = 4$										
w	0.02	0.05	0.10	0.20	0.30	0.50	0.70	1.00	1.40	1.80
h_1	.01188	.02919	.05682	.10812	.15523	.24072	.31928	.43356	.60035	.82855
h_{12}	.98812	.97081	.94318	.89188	.84477	.75928	.68072	.56644	.39965	.17145
h_2	.02903	.07261	.14544	.29269	.44367	.76547	1.1325	1.8480	3.6269	11.668
h_{23}	.98314	.95839	.91859	.84419	.77614	.65585	.55152	.41417	.24835	.08446
h_3	.02888	.07168	.14165	.27704	.40762	.66119	.91752	1.3512	2.2538	5.7481
h_{34}	.98826	.97163	.94624	.90243	.86563	.80598	.75799	.69757	.62486	.54688
h_4	.01174	.02837	.05376	.09757	.13437	.19402	.24201	.30243	.37514	.45312

$h_{n+1-i} = h_i$ $h_{n-i, n-i+1} = h_{i, i+1}$ $n = 5$										
w	0.02	0.05	0.10	0.20	0.30	0.50	0.70	1.00	1.40	1.80
h_1	.00952	.02316	.04433	.08186	.11442	.16931	.21549	.27639	.35405	.44321
h_{12}	.99048	.97684	.95567	.91814	.88558	.83069	.78451	.72361	.64595	.55679
h_2	.02527	.06262	.12353	.24107	.35424	.57404	.79624	1.1708	1.9372	4.8059
h_{23}	.98444	.96166	.92511	.85682	.79412	.68195	.58247	.44722	.27568	.09597
h_3	.03142	.07858	.15740	.31677	.48016	.82843	1.2256	2.0000	3.9252	12.628

TABLE V (Cont'd.)

$n=6$										
w	0.02	0.05	0.10	0.20	0.30	0.50	0.70	1.00	1.40	1.80
h_1	.00800	.01954	.03767	.07043	.09953	.15005	.19407	.25432	.33508	.43365
h_{12}	.99200	.98046	.96233	.92957	.90047	.84995	.80593	.74568	.66492	.56635
h_2	.02208	.05474	.10802	.21104	.31056	.50475	.70206	1.0352	1.7101	4.1589
h_{23}	.98605	.96560	.93271	.87094	.81371	.70965	.61512	.48241	.30570	.10915
h_3	.03035	.07591	.15205	.30600	.46383	.80026	1.1839	1.9320	3.7918	12.198
h_{34}	.98386	.96015	.92186	.84966	.78275	.66229	.55585	.41418	.24405	.08125
h_4	.03028	.07548	.15028	.29852	.44619	.74686	1.0698	1.6588	3.0271	9.0807
h_{45}	.98612	.96600	.93427	.87672	.82575	.73817	.66313	.56152	.42104	.21163
h_5	.02195	.05393	.10487	.19904	.28478	.43837	.57767	.77954	1.0829	1.5541
h_{56}	.99206	.98083	.96370	.93420	.95948	.86952	.83747	.79724	.74902	.69752
h_6	.00794	.01917	.03630	.06580	.09052	.13048	.16253	.20276	.25098	.30248
$h_{n+1-i} = h_i$ $h_{n-i, n-i+1} = h_{i, i+1}$ $n=7$										
w	0.02	0.05	0.10	0.20	0.30	0.50	0.70	1.00	1.40	1.80
h_1	.00685	.01661	.03169	.05818	.08090	.11869	.14999	.19058	.24125	.29798
h_{12}	.99315	.98339	.96831	.94182	.91910	.88131	.85001	.80942	.75875	.70202
h_2	.01937	.04765	.09285	.17699	.25438	.39508	.52510	.71758	1.0157	1.5029
h_{23}	.98757	.96954	.94099	.88885	.84214	.76055	.68916	.59038	.45046	.23462
h_3	.02822	.07025	.13958	.27628	.41170	.68577	.97829	1.5076	2.7229	8.0488
h_{34}	.98443	.96157	.92473	.85539	.79115	.67512	.57160	.43146	.25826	.08695
h_4	.03142	.07858	.15740	.31677	.48016	.82843	1.2256	2.0000	3.9252	12.628
$n=8$										
w	0.02	0.05	0.10	0.20	0.30	0.50	0.70	1.00	1.40	1.80
h_1	.00602	.01466	.02812	.05214	.07313	.10885	.13924	.17977	.23224	.29361
h_{12}	.99398	.98534	.97188	.94786	.92687	.89115	.86076	.82023	.76776	.70639
h_2	.01727	.04254	.08307	.15905	.22956	.35942	.48135	.66517	.95733	1.4556
h_{23}	.98882	.97256	.94672	.89916	.85610	.77972	.71162	.61550	.47636	.25569
h_3	.02603	.06477	.12860	.25423	.37847	.62940	.89655	1.3777	2.4711	7.2093
h_{34}	.98533	.96378	.92904	.86353	.80261	.69165	.59130	.45262	.27564	.09401
h_4	.03082	.07709	.15441	.31075	.47103	.81269	1.2023	1.9620	3.8506	12.388
h_{45}	.98411	.96074	.92297	.85159	.78515	.66474	.55753	.41414	.24235	.08012
h_5	.03078	.07684	.15340	.30645	.46079	.78107	1.1337	1.7952	3.3855	10.557
h_{56}	.98536	.96401	.92995	.86701	.81000	.70966	.62182	.50170	.33927	.13396
h_6	.02594	.06420	.12633	.24514	.35809	.57285	.78342	1.1230	1.7600	3.7771
h_{67}	.98887	.97287	.94791	.90341	.86473	.79964	.74509	.67322	.57968	.46254
h_7	.01718	.04200	.08103	.15159	.21417	.32251	.41667	.54595	.72282	.95311
h_{78}	.99402	.98555	.97265	.95045	.93186	.90184	.87778	.84763	.81153	.77303
h_8	.00598	.01445	.02735	.04955	.06814	.09816	.12222	.15237	.18847	.22697

TABLE V (Cont'd.)

$h_{n+1-i} = h_i$ $h_{n-i, n-i+1} = h_{i, i+1}$ $n = 9$										
w	0.02	0.05	0.10	0.20	0.30	0.50	0.70	1.00	1.40	1.80
h_1	.00534	.01294	.02464	.04509	.06253	.09131	.11496	.14536	.18291	.22441
h_{12}	.99466	.98706	.97536	.95491	.93747	.90869	.88504	.85464	.81709	.77559
h_2	.01549	.03792	.07336	.13803	.19606	.29812	.38853	.51533	.69390	.93598
h_{23}	.98991	.97536	.95255	.91152	.87540	.81360	.76075	.68961	.59453	.47118
h_3	.02390	.05913	.11633	.22581	.33008	.52914	.72539	1.0434	1.6401	3.4877
h_{34}	.98634	.96641	.93460	.87569	.82208	.72688	.64241	.52494	.36200	.14705
h_4	.02946	.07347	.14645	.29172	.43750	.73829	1.0674	1.6811	3.1466	9.7400
h_{45}	.98443	.96154	.92461	.85489	.79006	.67242	.56714	.42484	.25113	.08357
h_5	.03142	.07858	.15740	.31677	.48016	.82843	1.2256	2.0000	3.9252	12.628

$n = 10$										
w	0.02	0.05	0.10	0.20	0.30	0.50	0.70	1.00	1.40	1.80
h_1	.00482	.01172	.02242	.04136	.05776	.08536	.10852	.13897	.17767	.22192
h_{12}	.99518	.98828	.97758	.95864	.94224	.91464	.89148	.86103	.82233	.77808
h_2	.01408	.03452	.06697	.12664	.18072	.27714	.36400	.48808	.66738	.91954
h_{23}	.99079	.97748	.95650	.91845	.88459	.82578	.77460	.70438	.60822	.47947
h_3	.02206	.05459	.10744	.20877	.30555	.49116	.67521	.97511	1.5387	3.2518
h_{34}	.98728	.96870	.93899	.88378	.83327	.74273	.66136	.54641	.38336	.15995
h_4	.02793	.06962	.13864	.27575	.41298	.69522	1.0029	1.5741	2.9291	9.0044
h_{45}	.98500	.96295	.92736	.86012	.79748	.68326	.58015	.43883	.26242	.08802
h_5	.03103	.07760	.15544	.31281	.47416	.81807	1.2103	1.9750	3.8762	12.470
h_{56}	.98423	.96103	.92353	.85256	.78639	.66607	.55852	.41428	.24163	.07963
h_6	.03100	.07744	.15479	.31006	.46756	.79749	1.1652	1.8645	3.5691	11.281
h_{67}	.98502	.96310	.92796	.86242	.80240	.69543	.60086	.47159	.30189	.10912
h_7	.02787	.06921	.13701	.26908	.39768	.65095	.91117	1.3585	2.3066	6.1430
h_{78}	.98731	.96893	.93990	.88714	.84024	.75933	.68974	.59542	.46577	.26800
h_8	.02198	.05411	.10558	.20165	.29023	.45164	.60124	.82427	1.1784	1.8176
h_{89}	.99083	.97770	.95735	.92142	.89051	.83908	.79651	.74123	.67127	.58906
h_9	.01401	.03415	.06558	.12172	.17080	.25425	.32522	.42017	.54479	.69615
h_{910}	.99520	.98842	.97807	.96028	.94539	.92136	.90211	.87799	.84913	.81839
h_{10}	.00480	.01158	.02193	.03972	.05461	.07864	.09789	.12201	.15087	.18161

TABLE VI
 h VALUES FOR 0.001-DB RIPPLE CHEBYSHEV FILTERS HAVING h_0 AND $h_{n+1} = 1.0$ AND VARIOUS STOP-BAND 0.001-DB FRACTIONAL BANDWIDTHS w

$h_{n+1-i} = h_i$ $h_{n-i, n-i+1} = h_{i, i+1}$ $n = 3$										
w	0.02	0.05	0.10	0.20	0.30	0.50	0.70	1.00	1.40	1.80
h_1	.00638	.01581	.03117	.06081	.08937	.14481	.20033	.29018	.44516	.72075
h_{12}	.99362	.98419	.96883	.93919	.91063	.85519	.79967	.70982	.55474	.27925
h_2	.01141	.02854	.05718	.11507	.17442	.30093	.44520	.72650	1.4258	4.5869

$h_{n+1-i} = h_i$ $h_{n-i, n-i+1} = h_{i, i+1}$ $n = 5$										
w	0.02	0.05	0.10	0.20	0.30	0.50	0.70	1.00	1.40	1.80
h_1	.00838	.02045	.03935	.07335	.10336	.15507	.19972	.26024	.34027	.43633
h_{12}	.99162	.97955	.96065	.92665	.89664	.84493	.80028	.73976	.65973	.56367
h_2	.01761	.04399	.08790	.17564	.26378	.44423	.63709	.97773	1.6928	4.2897
h_{23}	.99091	.97733	.95481	.91031	.86651	.78056	.69547	.56497	.37232	.13682
h_3	.02058	.05148	.10311	.20752	.31455	.54270	.80289	1.3102	2.5714	8.2723

$h_{n+1-i} = h_i$ $h_{n-i, n-i+1} = h_{i, i+1}$ $n = 7$										
w	0.02	0.05	0.10	0.20	0.30	0.50	0.70	1.00	1.40	1.80
h_1	.00901	.02165	.04071	.07292	.09932	.14091	.17334	.21291	.25876	.30593
h_{12}	.99099	.97835	.95929	.92708	.90068	.85909	.82666	.78709	.74124	.69407
h_2	.01946	.04845	.09610	.18833	.27610	.43939	.59087	.81031	1.1302	1.5942
h_{23}	.98972	.97418	.94822	.89704	.84784	.75682	.67444	.56048	.40607	.19396
h_3	.02398	.05969	.11869	.23597	.35405	.60021	.87324	1.3866	2.6015	7.9470
h_{34}	.98635	.96645	.93468	.87526	.82001	.71775	.62211	.48393	.29840	.10209
h_4	.02430	.06078	.12174	.24500	.37138	.64075	.94794	1.5469	3.0360	9.7667

$h_{n+1-i} = h_i$ $h_{n-i, n-i+1} = h_{i, i+1}$ $n = 9$										
w	0.02	0.05	0.10	0.20	0.30	0.50	0.70	1.00	1.40	1.80
h_1	.00923	.02187	.04027	.06968	.09235	.12566	.14981	.17733	.20681	.23476
h_{12}	.99077	.97813	.95973	.93032	.90765	.87434	.85019	.82267	.79319	.76524
h_2	.02019	.05008	.09844	.18828	.26846	.40372	.51454	.65423	.82239	1.0074
h_{23}	.98921	.97279	.94537	.89269	.84481	.76413	.69936	.62054	.53031	.43516
h_3	.02510	.06219	.12298	.24290	.36283	.60810	.86733	1.3108	2.1873	5.1708
h_{34}	.98574	.96548	.93404	.87694	.82439	.72647	.63417	.50132	.32255	.11839
h_4	.02623	.06584	.13247	.26793	.40677	.70042	1.0308	1.6643	3.2129	10.190
h_{45}	.98839	.97084	.94141	.88286	.82568	.71667	.61416	.46864	.28138	.09425
h_5	.02740	.06854	.13730	.27630	.41882	.72260	1.0690	1.7445	3.4238	11.014

TABLE VII
 h VALUES FOR 0.001-DB RIPPLE CHEBYSHEV FILTERS HAVING h_0 AND $h_{n+1}=1.0$ AND VARIOUS STOP-BAND 0.01-DB FRACTIONAL BANDWIDTHS w

$$\begin{aligned} h_{n+1-i} &= h_i \\ h_{n-i, n-i+1} &= h_{i, i+1} \\ n &= 3 \end{aligned}$$

w	0.02	0.05	0.10	0.20	0.30	0.50	0.70	1.00	1.40	1.80
h_1	.00979	.02412	.04718	.09061	.13122	.20672	.27825	.38616	.55251	.79887
h_{12}	.99021	.97588	.95282	.90939	.86878	.79328	.72175	.61384	.44749	.20113
h_2	.01524	.03812	.07636	.15366	.23292	.40187	.59454	.97020	1.9041	6.1256

$$\begin{aligned} h_{n+1-i} &= h_i \\ h_{n-i, n-i+1} &= h_{i, i+1} \\ n &= 5 \end{aligned}$$

w	0.02	0.05	0.10	0.20	0.30	0.50	0.70	1.00	1.40	1.80
h_1	.01161	.02805	.05319	.09664	.13320	.19260	.24052	.30100	.37401	.45261
h_{12}	.98839	.97195	.94681	.90336	.86680	.80740	.75948	.69900	.62599	.54739
h_2	.02056	.05164	.10397	.21045	.31921	.54520	.78913	1.2228	2.1469	5.6562
h_{23}	.99132	.97808	.95555	.90934	.86236	.76807	.67433	.53354	.33743	.11888
h_3	.02478	.06197	.12414	.24982	.37868	.65334	.96657	1.5773	3.0956	9.9587

$$\begin{aligned} h_{n+1-i} &= h_i \\ h_{n-i, n-i+1} &= h_{i, i+1} \\ n &= 7 \end{aligned}$$

w	0.02	0.05	0.10	0.20	0.30	0.50	0.70	1.00	1.40	1.80
h_1	.01207	.02862	.05279	.09155	.12155	.16585	.19811	.23503	.27477	.31262
h_{12}	.98793	.97138	.94721	.90845	.87845	.83415	.80189	.76497	.72523	.68738
h_2	.02200	.05533	.11126	.22183	.32791	.52278	.69744	.93756	1.2609	1.6795
h_{23}	.99037	.97503	.94776	.89024	.83269	.72539	.63067	.50593	.34905	.15468
h_3	.02723	.06733	.13267	.26030	.38734	.65143	.94608	1.5067	2.8562	8.8674
h_{34}	.98228	.95702	.91790	.84816	.78635	.67715	.57907	.44224	.26659	.08970
h_4	.02565	.06416	.12853	.25866	.39207	.67645	1.0008	1.6331	3.2051	10.311

$$\begin{aligned} h_{n+1-i} &= h_i \\ h_{n-i, n-i+1} &= h_{i, i+1} \\ n &= 9 \end{aligned}$$

w	0.02	0.05	0.10	0.20	0.30	0.50	0.70	1.00	1.40	1.80
h_1	.01217	.02837	.05102	.08509	.10971	.14359	.16656	.19128	.21619	.23841
h_{12}	.98783	.97163	.94898	.91491	.89029	.85641	.83344	.80872	.78381	.76159
h_2	.02259	.05684	.11360	.22064	.31526	.46864	.58677	.72572	.87959	1.0341
h_{23}	.98988	.97323	.94321	.88189	.82530	.73280	.66300	.58408	.50127	.42157
h_3	.02787	.06829	.13343	.26147	.39197	.66934	.97416	1.5102	2.5978	6.4690
h_{34}	.98214	.95809	.92335	.86551	.81485	.71965	.62644	.48889	.30576	.10825
h_4	.02717	.06883	.14022	.28834	.44210	.76889	1.1369	1.8437	3.5788	11.409
h_{45}	.99148	.97761	.95229	.89691	.83922	.72540	.61750	.46611	.27621	.09175
h_5	.02994	.07488	.14998	.30183	.45752	.78937	1.1678	1.9057	3.7401	12.032

TABLE VIII
 h VALUES FOR 0.1 DB RIPPLE CHEBYSHEV FILTERS HAVING h_0 AND $h_{n+1}=1.0$ AND VARIOUS STOP-BAND 0.1-DB FRACTIONAL BANDWIDTHS w

$\begin{aligned} h_{n+1-i} &= h_i \\ h_{n-i, n-i+1} &= h_{i, i+1} \\ n &= 3 \end{aligned}$										
w	0.02	0.05	0.10	0.20	0.30	0.50	0.70	1.00	1.40	1.80
h_1	.01595	.03895	.07509	.14043	.19849	.29936	.38729	.50775	.66936	.86689
h_{12}	.98405	.96105	.92491	.85957	.80151	.70064	.61271	.49225	.33064	.13311
h_2	.01802	.04508	.09030	.18173	.27547	.47527	.70313	1.1474	2.2519	7.2444

$\begin{aligned} h_{n+1-i} &= h_i \\ h_{n-i, n-i+1} &= h_{i, i+1} \\ n &= 5 \end{aligned}$										
w	0.02	0.05	0.10	0.20	0.30	0.50	0.70	1.00	1.40	1.80
h_1	.01739	.04133	.07645	.13323	.17755	.24360	.29214	.34819	.40911	.46770
h_{12}	.98261	.95867	.92355	.86677	.82245	.75640	.70786	.65181	.59089	.53230
h_2	.02184	.05570	.11477	.24131	.37710	.67240	1.0023	1.6024	2.9173	8.1267
h_{23}	.99617	.98936	.97548	.94033	.89822	.80262	.70050	.54436	.33347	.11391
h_3	.03103	.07760	.15544	.31281	.47416	.81807	1.2103	1.9750	3.8762	12.470

$\begin{aligned} h_{n+1-i} &= h_i \\ h_{n-i, n-i+1} &= h_{i, i+1} \\ n &= 7 \end{aligned}$										
w	0.02	0.05	0.10	0.20	0.30	0.50	0.70	1.00	1.40	1.80
h_1	.01758	.04073	.07269	.11982	.15322	.19825	.22823	.25997	.29143	.31907
h_{12}	.98242	.95927	.92731	.88018	.84678	.80175	.77177	.74003	.70857	.68993
h_2	.02298	.05948	.12424	.26024	.39467	.63830	.84496	1.1064	1.4196	1.7691
h_{23}	.99524	.98487	.96081	.89735	.82625	.69022	.57492	.43511	.27905	.11263
h_3	.03215	.07781	.14880	.27950	.40417	.65899	.94530	1.5008	2.8685	9.0265
h_{34}	.97199	.93375	.87813	.78827	.71650	.60184	.50671	.38050	.22532	.07492
h_4	.02472	.06182	.12382	.24919	.37772	.65168	.96412	1.5733	3.0878	9.9334

$\begin{aligned} h_{n+1-i} &= h_i \\ h_{n-i, n-i+1} &= h_{i, i+1} \\ n &= 9 \end{aligned}$										
w	0.02	0.05	0.10	0.20	0.30	0.50	0.70	1.00	1.40	1.80
h_1	.01747	.03954	.06836	.10775	.13362	.16613	.18640	.20677	.22593	.24199
h_{12}	.98253	.96046	.93164	.89225	.86638	.83387	.81360	.79323	.77407	.75801
h_2	.02359	.06174	.12920	.26248	.37938	.55700	.68092	.81264	.94328	1.0612
h_{23}	.99451	.98120	.94999	.87487	.80297	.69180	.61627	.53992	.46890	.40778
h_3	.03198	.07557	.14179	.26884	.40394	.71890	1.0976	1.8044	3.3008	8.8634
h_{34}	.97290	.94013	.89990	.84795	.81051	.73782	.65347	.51178	.31400	.10796
h_4	.02622	.06837	.14470	.31307	.49447	.88463	1.3225	2.1595	4.2141	13.488
h_{45}	1.0031	1.0038	.99651	.96023	.90822	.78921	.66997	.50190	.29457	.09727
h_5	.03464	.08665	.17356	.34929	.52945	.91347	1.3514	2.2053	4.3281	13.924

TABLE IX
 h VALUES FOR 0.5-DB RIPPLE CHEBYSHEV FILTERS HAVING h_0 AND $h_{n+1} = 1.0$ AND VARIOUS STOP-BAND 0.5-DB FRACTIONAL BANDWIDTHS

$$\begin{aligned} h_{n+1-i} &= h_i \\ h_{n-i, n-i+1} &= h_{i, i+1} \\ n &= 3 \end{aligned}$$

w	0.02	0.05	0.10	0.20	0.30	0.50	0.70	1.00	1.40	1.80
h_1	.02446	.05902	.11161	.20181	.27706	.39803	.49449	.61484	.75804	.90974
h_{12}	.97554	.94098	.88839	.79819	.72294	.60197	.50551	.38516	.24196	.09026
h_2	.01723	.04309	.08631	.17370	.26329	.45427	.67206	1.0967	2.1524	6.9243

$$\begin{aligned} h_{n+1-i} &= h_i \\ h_{n-i, n-i+1} &= h_{i, i+1} \\ n &= 5 \end{aligned}$$

w	0.02	0.05	0.10	0.20	0.30	0.50	0.70	1.00	1.40	1.80
h_1	.02543	.05910	.10583	.17540	.22513	.29280	.33822	.38666	.43503	.47782
h_{12}	.97457	.94090	.89417	.82460	.77487	.70720	.66178	.61334	.56497	.52218
h_2	.01997	.05231	.11219	.25189	.41416	.79356	1.2404	2.0805	3.9695	11.643
h_{23}	1.0068	1.0147	1.0221	1.0183	.99536	.91300	.80488	.62532	.37833	.12742
h_3	.03991	.09983	.19997	.40242	.60999	1.0524	1.5570	2.5408	4.9866	16.042

$$\begin{aligned} h_{n+1-i} &= h_i \\ h_{n-i, n-i+1} &= h_{i, i+1} \\ n &= 7 \end{aligned}$$

w	0.02	0.05	0.10	0.20	0.30	0.50	0.70	1.00	1.40	1.80
h_1	.02522	.05665	.09695	.15073	.18526	.22781	.25385	.27967	.30365	.32350
h_{12}	.97478	.94335	.90305	.84927	.81474	.77219	.74615	.72033	.69635	.67650
h_2	.02110	.05738	.12787	.29063	.45917	.75958	.99691	1.2688	1.5559	1.8356
h_{23}	1.0055	1.0065	.99241	.92515	.83515	.65888	.51872	.36589	.21727	.08087
h_3	.03926	.09135	.16530	.28646	.39263	.60264	.84106	1.3163	2.5161	7.9686
h_{34}	.95394	.89420	.81345	.69706	.61514	.50011	.41440	.30731	.18023	.05959
h_4	.02112	.05282	.10581	.21293	.32276	.55687	.82385	1.3444	2.6385	8.4882

$$\begin{aligned} h_{n+1-i} &= h_i \\ h_{n-i, n-i+1} &= h_{i, i+1} \\ n &= 9 \end{aligned}$$

w	0.02	0.05	0.10	0.20	0.30	0.50	0.70	1.00	1.40	1.80
h_1	.02477	.05394	.08882	.13146	.15675	.18590	.20275	.21876	.23304	.24447
h_{12}	.97523	.94606	.91118	.86854	.84325	.81410	.79725	.78124	.76696	.75553
h_2	.02194	.06137	.13866	.30266	.44530	.64487	.76870	.88706	.99296	1.0804
h_{23}	1.0041	.99909	.96934	.87568	.78303	.65106	.57233	.50185	.44351	.39791
h_3	.03777	.08345	.14515	.25794	.38684	.73311	1.2051	2.1578	4.2675	12.327
h_{34}	.95679	.91017	.86294	.82462	.81376	.79130	.73297	.59119	.36317	.12321
h_4	.02311	.06351	.14413	.34184	.56918	1.0698	1.6282	2.6823	5.2546	16.852
h_{45}	1.0257	1.0560	1.0875	1.0970	1.0626	.93991	.80021	.59803	.34942	.11505
h_5	.04279	.10702	.21438	.43142	.65395	1.1283	1.6692	2.7239	5.3460	17.198